Wire Scanners and Bunch Length Detectors for Beam Size and Emittance Measurements in the Linac Upgrade Transition Section

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LU-180
April 15, 1992

1 Introduction

One way to measure the beam emittances in the transverse plane is to use a system of wire scanners. The emittance is reconstructed from position profiles at each scanner, which are projections of the emittance onto the position axis. This method of calculation assumes a Gaussian beam distribution, a good approximation in general. With three unknown Twiss parameters, α, β, and γ the minimum number of profiles needed is three, although in general more than three scanners are used to account for hardware failure problems and to minimize the rms errors in the measurements. Problems such as wire heating, wire vibrations, can lead to either a failure or errorneous output. For such reasons it is common practice in many laboratories (SLC, Moscow Meson Linac, BNL) to use more than three wire scanners for beam profile measurements . The beam size in the transition section is about 40 times bigger than the SLC beam size making it relatively easier to perform the measurement. Previous experience with the existing three wire scanners in the 200 MeV line suggests that three wire scanners in the transition section should be reliable enough. However to minimize the errors in the calculated beam emittance, and to add a backup in case of a hardware failure, one can also use a properly placed quadrupole and a wire scanner to measure the beam emittance [4].

In the simplest case the three wire scanners are separated by drift spaces in a field free region like in the 200 MeV line. This method requires knowledge of the transfer matrices between the three or more scanners. For the simple situation this calculation can be easily carried out and optimized to achieve a good accurate measurement of the emittance. The calculation becomes more complicated when other elements, such as quadrupoles and coupled cavities, are included in the drift region. This is the case in the transition section. For this one can use computer codes like TRACE3D [2] or TRANSPORT [3] to compute the transfer matrices and emittances. These codes take into account space charge effects where now the transfer matrices depend on the emittance and the calculation of emittance is done iteratively.

This note looks first at the lattice characteristics of the transition section. We then calculate the nominal beam sizes at the proposed wire scanner locations and the nominal emittances. Next the sensitivity coefficients of the emittance around the nominal values are calculated for small variations in the measured beam size. The beam size sensitivity for small variations in the quadrupole field gradients are also examined. The overall rms error in the emittance is derived using the law of propagation of errors. Issues related to the hardware performance and design characteristics of the wire scanners, and the bunch length detectors are not within the scope of this paper. Finally the effect of relocating the first buncher (16 cavities) and the quadrupoles will have on the beam is discussed.

2 Beam Characteristics in the Transition Section

The beam envelopes in both transverse and longitudinal planes are shown in Fig. 1. Figures 2, and 3 show the Twiss parameters and phase advances from element to element as calculated by TRACE3D. The values given are at the end of each element hence the non zero phase advance at element number zero in Fig. 3. Looking at Fig. 1 the lattice characteristics of the transition section are very similar to two "FODO" cells. Transverse matching of the beam entering the first section is achieved by a set of BPMs, steering coils, and wire scanners. Fine longitudinal tuning, if necessary, is achieved by adjusting the gradient of the second smaller buncher (6 cavities) which serves as a vernier [1].

3 Wire Scanners For Transverse Emittance Measurements

A code like TRACE3D follows the transformation of the ellipse through various optical elements of a beam line. Given the transfer matrix $[R]$ of a beam line, the initial sigma matrix $[\sigma]_i$ that defines the ellipse parameters at the begining of the selected line, the final sigma matrix $[\sigma]_f$ is calculated from the matrix multiplication:

$$[\sigma]_f = [R][\sigma]_i[R]^T \tag{1}$$

The elements of the sigma matrix may be interpreted as second moments of the beam about its center of gravity. The square roots of the diagonal terms of the sigma matrix are a measure of the "beam size" in each coordinate. The off-diagonal terms in the sigma matrix determine the orientation of the ellipse. The elements of the sigma matrix are related to the Twiss parameters α,β, and γ by:

$$[\sigma] = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} \tag{2}$$

where ϵ is the beam emittance and the $Det\,[\sigma] = \epsilon$. At each wire scanner two beam profiles, in the transverse and horizontal planes, are measured. Each profile can be fit to a Gaussian distribution, from which the root mean square σ_{rms} is calculated. This latter is simply the second moment of the beam assuming a Gaussian distribution. It is also possible to use the integral definition and calculate directly the second moment of the beam profile distribution without using a Gaussian distribution fit. These two methods of calculations should be available in a general emittance program software. Different definitions are used to determine the emittance boundary. The emittance of 39% of the beam is related to the rms of the beam by

$$\epsilon_{rms} = (\pi)\frac{\sigma_{rms}^2}{\beta} \tag{3}$$

For 86% of the beam this relation becomes

$$\epsilon = (\pi)\frac{(2\sigma_{rms})^2}{\beta} = 4\epsilon_{rms} \tag{4}$$

and for 95% of the beam it is

$$\epsilon = (\pi)\frac{(\sqrt{6}\sigma_{rms})^2}{\beta} = 6\epsilon_{rms}. \tag{5}$$

Here β is the value at the corresponding wire scanner where the measurement is made. For an accelerated beam this emittance corresponds to a normalized emittance ϵ_N also equal to $\beta\gamma\epsilon$ where $\beta\gamma$ is proportional to the momentum or, above few GeV, to the energy of the proton. For a proton machine the emittance boundary is usually chosen to include 95% of a Gaussian beam or $\sqrt{6}\sigma_{rms}$. The square roots of the σ matrix in TRACE3D represent the maximum extent of the beam in the various coordinate systems. For 95% of the beam this maximum half-width corresponds to $\sqrt{6}\sigma_{rms}$.

3.1 Basic Equations

Three equations are needed to determine the elements of the 2x2 symmetric sigma matrix where $\sigma_{12} = \sigma_{21}$. Given the transfer matrix between three wire scanners locations, labeled respectively as a,b,c, and the beam rms value at each location (σ_{ii}) one can write the following two set of equations:

$$\begin{aligned} \sigma^b_{11} &= R^2_{11}\sigma^a_{11} + 2R_{11}R_{12}\sigma^a_{12} + R^2_{12}\sigma^a_{22} \\ \sigma^c_{11} &= S^2_{11}\sigma^a_{11} + 2S_{11}R_{12}\sigma^a_{12} + S^2_{12}\sigma^a_{22} \end{aligned} \tag{6}$$

The two unknowns σ^a_{12} and σ^a_{22} are derived from the above equations. These equations assume no coupling between the horizontal and vertical planes. In case of coupling one needs to solve four equations (two equations in each plane) with four unknowns. It should be noted that for a "FODO" cells like lattice the elements R_{11}, R_{12} vary like $\cos(\phi)$ and $\sin(\phi)$ respectively. The coefficients in eqs (6) take respectively the form of $\cos^2\phi$, $\sin 2\phi$ and $\sin^2\phi$. In an optimized setup one would like to have a phase advance $\phi = 45^o$ between each of the three scanners.

3.2 The Code TRACE3D

The option "l" in TRACE3D provides a mechanism to determine the emittances in the two transverse planes given the beam sizes at three locations. The user has to first enter the location of three designated wire scanners in element number and not real distances. The user then specify the beam half-width, in mm, at each location. This latter has the same meaning as $\sqrt{6}\sigma_{rms}$ for 95% of a Gaussian beam. When space-charge forces are included TRACE3D assumes a charge distribution that is a uniform ellipsoid and has the same second moments for the spatial coordinates as the beam we are trying to simulate. Given an rms value this implies a second moment equal to $\sqrt{5}\sigma_{rms}$ and an emittance equal to $5\epsilon_{rms}$ (see Appendix D of [2]). The code then performs the calculations described in eq. (6) to determine the elements of the sigma matrix and to derive the emittance. The units of emittance are in π mm-mrad in the transverse planes and π deg.KeV in the

	XHW (mm)	YHW (mm)	ZHW (mm)
Loc 1	9.48	4.50	9.10
Loc 2	3.81	8.23	7.35
Loc 3	6.02	6.49	5.26

Table 1: Nominal half-beam widths

longitudinal plane. This corresponds to a beam half-width longitudinally in degrees which can be converted to units of mm by:

$$\Delta z = \frac{\beta c}{360 f} \Delta \phi \tag{7}$$

where β is the relativistic value (not to be confused with the Twiss parameter), c the speed of light and f the resonant frequency of the cavity.

Initial calculations with our existing version of TRACE3D have revealed a discrepency in the results using the elements of the transfer matrix **R** and the elements of the σ matrix in the longitudinal phase space (z,dp/p) plane. In other words the relation $\sigma_{\mathbf{f}} = \mathbf{R}\sigma_{\mathbf{i}}\mathbf{R}^{\mathbf{T}}$ was not satisfied in that plane. After analyzing the TRACE3D source code it was discovered that the relativistic γ^2 was missing from the calculation of the accumulated **R**, referred to as **RM** in TRACE3D. This calculation is performed in a subroutine called **DTRANS** whenever the chromaticity is turned on (ICHROM=1). The same bug was also detected at the SSC. The proper modification to the subroutine was made to correct the source of error. A new version of TRACE3D will be installed in the near future.

4 Transverse Beam Calculations

Three beam profile monitors are placed next to quads element number 5,9, and 13 respectively, as shown in Fig. 1. The nominal beam half-widths at each location and in each plane, as calculated by TRACE3D with space charge turned on, are given in Table 1. Throughout the transition section these values vary as follow:

- $3.81 \leq$ XHW (mm) ≤ 9.51
- $4.38 \leq$ YHW (mm) ≤ 10.50

- $4.67 \leq$ ZHW (mm) ≤ 9.10

The smallest nominal beam size to be measured is about 3.81 mm. Here, the relativistic $\beta = 0.457$ at 116.54 MeV and the frequency of the coupled cavities is $f = 805MHz$. The half-beam width at each location is also equal to $\sqrt{\beta\epsilon}$, β being the Twiss parameter at that location.

4.1 Sensitivity to Small Changes in Transverse Beam Size

The emittance sensitivity to changes in the measured transverse half-beam widths, XHW and YHW, around the nominal values, is independently examined for each wire scanner. These changes can be due to an emittance growth, a beta mismatch or simply because of mechanical errors. The question of resolving a beta mismatch from an emittance growth will be addressed in another paper. The dependence of the emittance on the half-beam widths can be represented by the relation:

$$\epsilon^2 = \sum_1^3 \alpha_{ij}\sigma_i\sigma_j \tag{8}$$

where $\sigma_i = XHW_i^2$ and the subscripts i, j refer to the wire scanner number. The coefficients α_{ij} are derived from the transfer [R] matrices. Equation (8) is a polynomial of second order in σ_i for $i = j$, and of fourth order in the half-beam width. Depending on the coefficients α_{ij} this dependence can locally look like a straight line or like a fourth order polynomial. The sensitivity coefficients of the emittance around the nominal values are summarized in Table 2.

$\frac{\partial\epsilon_x}{\partial xhw_1}\|_{9.48}$	$\frac{\partial\epsilon_x}{\partial xhw_2}\|_{3.81}$	$\frac{\partial\epsilon_x}{\partial xhw_3}\|_{6.02}$
1.77	-1.55	2.72
$\frac{\partial\epsilon_y}{\partial yhw_1}\|_{4.50}$	$\frac{\partial\epsilon_y}{\partial yhw_2}\|_{8.23}$	$\frac{\partial\epsilon_y}{\partial yhw_3}\|_{6.49}$
5.72	-4.94	6.52

Table 2: Sensitivity coefficients around nominal values in $\pi mrad$

We observe that the sensitivity coefficients per plane are more or less equal in magnitude. Based on the law of propagation of errors, the emittance has a variance equal to

$$\sigma_\epsilon^2 = \sum_1^3 \left(\frac{\partial \epsilon}{\partial xhw_i}\right)^2 \sigma_{xhw_i}^2 \tag{9}$$

It should be noted that eq. (9) assumes the errors to be statistically independent which may not be always true.

4.2 Sensitivity to Small Changes in Quadrupole Gradients

A change in quadrupole gradient will affect the transverse beam size and keep the beam emittance unchanged. Only the beam size at wire scanners located downward from the quadrupole will be affected by the changing gradient. For example, a change in the gradient of the quadrupole referred to as element 5 in Fig. 1 will affect the beam size at the second and third wire scanners. Some derived sensitivity coefficients are given in Table 3.

$\frac{\partial xhw_2}{\partial quad_1}\vert_{21.18}$	$\frac{\partial xhw_3}{\partial quad_1}\vert_{21.18}$	$\frac{\partial yhw_2}{\partial quad_1}\vert_{21.18}$	$\frac{\partial yhw_3}{\partial quad_1}\vert_{21.18}$
-0.624	0.0185	0.263	0.205
$\frac{\partial xhw_2}{\partial quad_2}\vert_{-13.52}$	$\frac{\partial xhw_3}{\partial quad_2}\vert_{-13.52}$	$\frac{\partial yhw_2}{\partial quad_2}\vert_{-13.52}$	$\frac{\partial yhw_3}{\partial quad_3}\vert_{-13.52}$
0.00	-0.138	0.00	0.458

Table 3: Sensitivity coefficients around nominal values in $\frac{mm}{T/m}$

The longitudinal beam size is not affected by the changing quadrupole gradient.

5 Bunch Length Detectors For Longitudinal Emittance Measurement

Similar to the transverse emittance calculations, the beam characteristics longitudinally can be derived using three bunch length detectors. But unlike the transverse case, TRACE3D has no option for the calculation of longitudinal emittance given three locations for longitudinal measurements. These calculations can be easily carried out if we assume the elements of the transfer matrix to be constant and independent of the charge distribution. Results of these calculations are given in the next section.

5.1 Sensitivity to Small Changes in Longitudinal Beam Size

The length of the BLD is designed to be about 20 cm making it hard to fit with other diagnostic elements at the begining of the transition section. For it to fit one needs to move the first buncher further down by at least 10 cm. The effect of relocating the buncher will have on the beam parameters is described in the next section. With three BLDs located as shown in Fig. 1 the following two algebraic equations, necessary to calculate the beam characteristics in the (z,dp/p), are derived:

$$\begin{aligned} \sigma_{56a} &= -0.45\sigma_{55a} + 0.94\sigma_{55b} - 0.39\sigma_{55c} \\ \sigma_{66a} &= 0.16\sigma_{55a} - 0.47\sigma_{55b} + 0.51\sigma_{55c} \end{aligned} \tag{10}$$

The longitudinal emittance at location "a" is given by

$$\epsilon_z^2 = \sigma_{55a}\sigma_{66a} - \sigma_{56a}\sigma_{65a} \tag{11}$$

where $\sigma_{65a} = \sigma_{56a}$. Here σ_{55i} refers to the longitudinal half-beam width. The sensitivity coefficients of the emittance around the nominal longitudinal half-beam widths are calculated using eqs. (10),(11). Again, these calculations are made by varying each measurement separately. Results are shown in Table 4. One notice the high sensitivity of the emittance to longitudinal beam size measurement at BLD # 2.

$\frac{\partial\epsilon_z}{\partial zhw_1}\|_{9.12}$	$\frac{\partial\epsilon_z}{\partial zhw_2}\|_{7.37}$	$\frac{\partial\epsilon_z}{\partial zhw_3}\|_{5.41}$
7.37	-24.4	14.14

Table 4: Sensitivity coefficients around nominal values in $\pi mrad$

6 Moving the Buncher and Quadrupoles

The output beam parameters, after the first buncher has been moved downard by 10 cm, and the two quadrupoles next to the two bunchers where the power feed is located have been moved by an extra 1 cm are shown in Fig. 4. To correct for the output longitudinal Twiss parameters one needs to increase the gradient on the small second buncher from 0.0076 MV/m to 0.6218 MV/m. By examing the convergence factors (measurement of how close the solution is from the desired set value based on linear interpolation method) in Table 5 one notice the high sensitivity of the small buncher gradient. This is a desired feature for fine tuning. The longitudinal out-

Convergence Factor in %	First Buncher Gradient	Small Buncher Gradient
34.9	1.8794	-0.0074
14.79	1.6569	-0.0065
14.79	1.6569	-0.0065
7.52	1.7904	-0.0070
3.17	1.9585	-0.0077
2.87	1.9464	-0.0077
3.04	1.8945	-0.0075
3.34	1.8840	-0.0074
2.61	1.9221	-0.0110
15.38	2.3341	-3.1503
0.15	1.9717	-0.5984
0.00	1.9713	-0.6218

Table 5: Adjusting the gradients on the two bunchers for longitudinal output beam tuning

put Twiss parameters for a 35% convergence factor are $\alpha = -0.072$ and $\beta = 0.0132 deg/KeV$ compared to the desired set values of $\alpha = 0.00$ and $\beta = 0.0131$.The difference between the desired and the calculated values is very small and may be at the noise level of the measurements. Therefore it is very likely that the gradients of the two bunchers will not need to be adjusted.

A quick review of the transition section layout has revealed that in addition to the 10 cm, gained by moving the first buncher, an extra 2 cm was still needed. For this a trim magnet shorter by 2 cm (4 cm instead of 6 cm) can be used. Calculation has shown that the change in the integrated quadrupole field strength is not very considerable (down to 1.5 mrad from 1.8 mrad).

7 Conclusions

In this note the beam size characteristics in the transition section have been calculated. The sensitivity coefficients of the emittance for variations in the measured beam size and of the beam size for variations in quadrupole gradient around the nominal values have been examined. Results have shown that the calculated transverse emittances are about equally sensitive to variations in the measured beam size. Longitudinal emittance calculations have revealed that the emittance is highly sensitive to variations in the beam size at the second BLD. The effect of moving the buncher and the quadrupoles will have on the beam, was also analyzed. Moving the first buncher by 10 cm to accomodate for a bunch length detector causes little harm to the beam. Another 2 cm is gained by using a trim magnet shorter by 2 cm without a considerable change in the integrated quadrupole field gradient. The extra 1 cm space needed by the two quadrupoles next to the power feed at the end of each buncher has almost no effect on the beam. A carefuly chosen pair of a quadrupole and a wire scanner can provide us with an extra set of measurement and can act as a back up in case of a hardware failure.

References

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[4] M. C. Ross, N. Phinney, G. Quickfall, H. Shoaee, and J. C. Sheppard,"Automated Emittance Measurement in the SLC", SLAC-87.

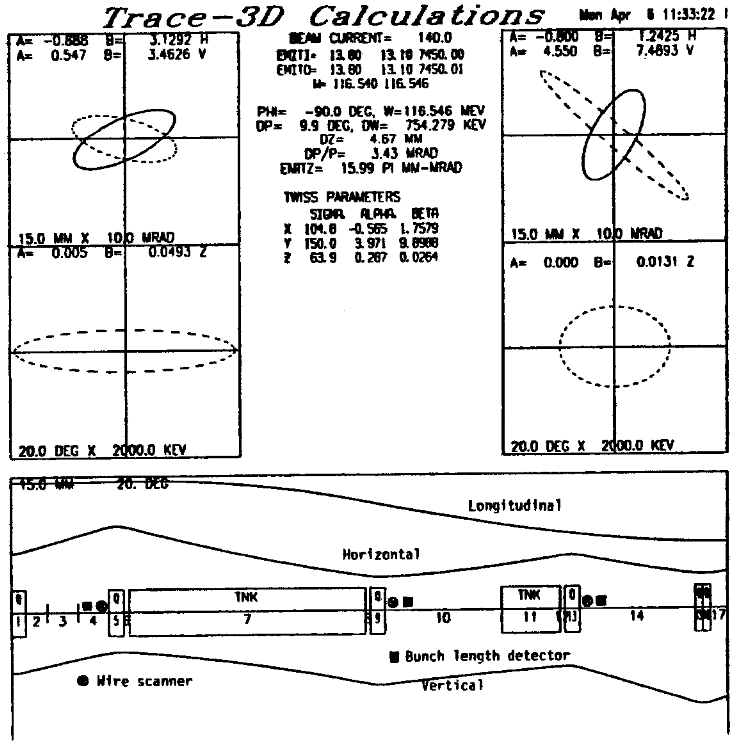

Figure 1: Beam parameters and locations of the wire scanners and bunch length detectors in transition section

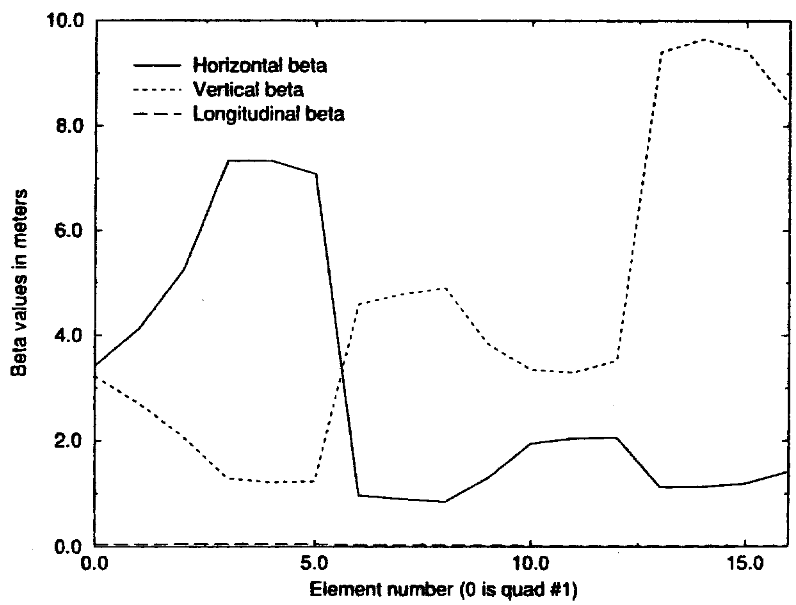

Figure 2: Twiss parameters in transition section

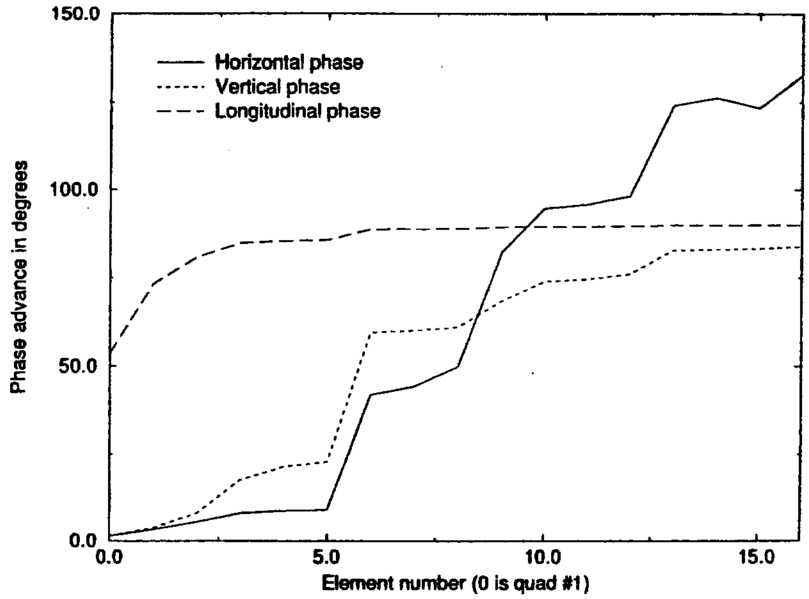

Figure 3: Phase advances in transition section

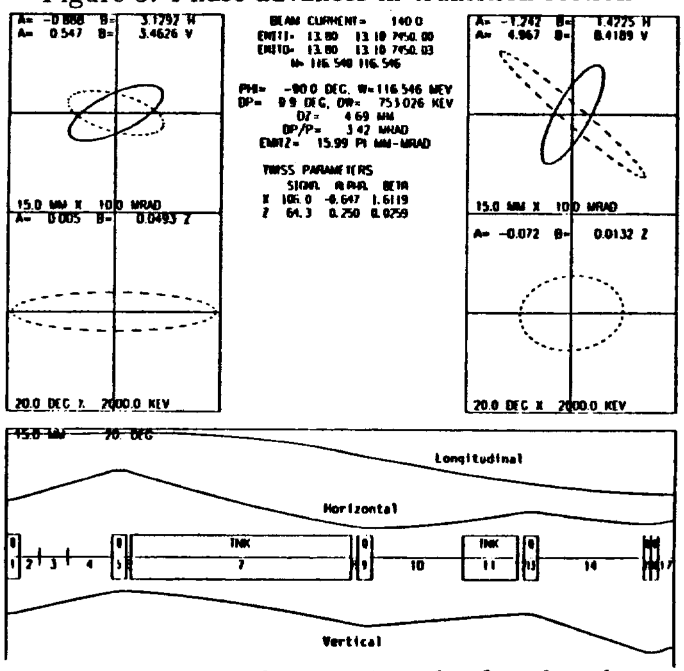

Figure 4: Beam parameters after moving the first buncher and the two quadrupoles